

**NAME:****Math 150 Practice Exam 1.1****Instructions:** WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK!

1. Given that  $\lim_{x \rightarrow 1} f(x) = 8$ ,  $\lim_{x \rightarrow 1} g(x) = 3$ , and  $\lim_{x \rightarrow 1} h(x) = 2$  find

a)  $\lim_{x \rightarrow 1} \frac{f(x)}{g(x) - h(x)}$  [5 pts]

b)  $\lim_{x \rightarrow 1} \sqrt[3]{f(x)g(x) + 3}$  [5 pts]

2. Use the squeeze theorem to evaluate  $\lim_{x \rightarrow 0^+} \sqrt{x} \sin\left(\frac{\pi}{x}\right)$  [10 pts]

3. Evaluate  $\lim_{x \rightarrow -\infty} \frac{\sqrt{16x^4 + 64x^2} + x^2}{2x^2 - 4}$  [10 pts]

4. Find an equation of the tangent line to the curve  $y = 4x^2 + 2x$  at the point  $a = -2$ . [10 pts]

5. Find the derivative of the function  $f(x) = \sqrt{x+2}$  using the definition of the derivative at the point  $a = 7$ . [10 pts]

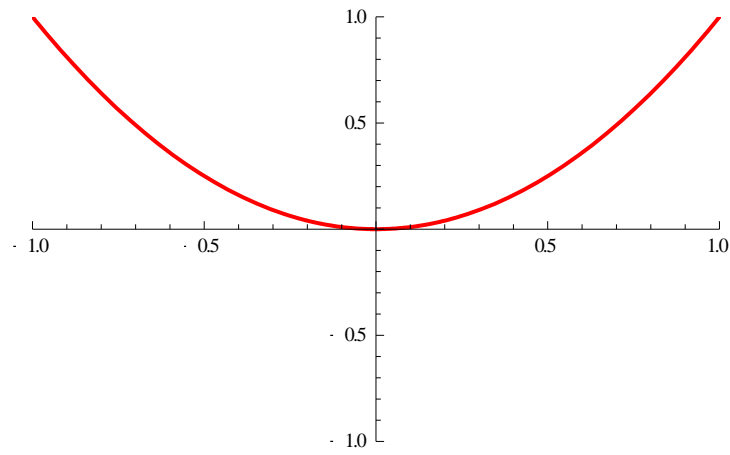
6. Evaluate  $\lim_{x \rightarrow 2} \frac{x^5 - 32}{x - 2}$  [10 pts]

7. Let  $f: [0, 1] \rightarrow (0, 1)$  be a continuous function such that  $0 < f(x) < 1$  for all  $x \in [0, 1]$ . Prove that the equation  $f(x) = x$  has a solution for at least one  $x \in [0, 1]$ . [10 pts]

8. Let  $a > 0$  be a positive real number. Define  $f(x) = \begin{cases} x & \text{if } x < a \\ 3x - 2 & \text{if } x \geq a \end{cases}$ .

What is the value of  $a$  if  $f$  is continuous on the entire real number line? [10 pts]

9. The graph of the function  $y = f(x)$  is displayed below



Draw the graph of  $y = f'(x)$ .

[10 pts]

10. Evaluate  $\lim_{x \rightarrow 0} \frac{\sin 3x}{x}$

[10 pts]

**Extra-Credit**

11. Prove by means of a delta-epsilon argument that  $\lim_{x \rightarrow 2} (3x - 1) = 5$   
[10 pts]
12. Establish the derivative product formula. Namely, show that  $(fg)' = f'g + fg'$   
[10 pts]